Noise Processing (Maximum Noise Fraction)

Assumptions:

1. Many of the spectrally important features that appear in hyperspectral data are subtle.
   - Noise in hyperspectral data can easily overwhelm subtle hyperspectral features.
   - Data are contiguous over a large spectral range, including very noisy spectral ranges.
   - There generally exist bands that are essentially noise, e.g., in strong water absorption bands, as well as bands that have relatively low S/N.
   - Groups of bands are frequently highly correlated. Large fluctuations in these regions can mask subtle features that occur within the bands in those regions
2. Careful noise removal will be required in order to enable extraction of useful information.

Task: Remove noise with minimal information loss

Most of the methods considered so far are not well-suited to the current task. We have been particularly concerned with procedures designed to deal with noise within individual bands (convolution filters, frequency domain filters). While these tools remain very important, the greater concern is with noise in the spectral domain. The only appropriate tool we have examined so far is Principal Components Analysis (PCA). That is problematic because the procedure equates variance with information and is based on the assumption that the data structure can be described by a multi-dimensional normal distribution. A related, much more appropriate procedure, the Maximum Noise Fraction (MNF) transformation, exists and is well worth considering.

Maximum Noise Fraction (MNF)

MNF is a linear transformation that consists of two separate PCA rotations and a noise whitening step:

- Use the noise covariance matrix to decorrelate and rescale the noise in the data (noise whitening). This results in transformed data in which the noise has unit variance and no band-to-band correlations.
- Perform a standard Principal Components transformation of the noise-whitened data.

The MNF transform[1], like the PCA transform, is an eigenvector procedure, but based on the covariance structure of the noise in the image data set. The MNF is much more effective at creating a set of images that is ordered according to image quality. This results in more reliable identification and elimination of noisy components and allows for targeted smoothing of those noisy components that are deemed to contain useful information.

The goal of the MNF transform is to select component in a way that maximizes the signal-to-noise ratio (rather than the information content). To do this one must know the covariance matrices of both the signal, Σ, and the noise, Σ_n. The signal covariance, Σ, is computed in the same way as is done for the PCA transform. The noise covariance, Σ_n, may be estimated using a flat-field image. Absent this, it is necessary to estimate the noise using in-scene statistics.

[1] Green, A.A., M. Berman, p. Switzer and M.D. Craig (1988) A transformation for ordering multispectral data in terms of image quality with implications for noise removal. IEEE Transactions on Geoscience and Rmote Sensing, 26(1):65-74.

Noise covariance determined from a flat-field image

An estimate of the noise covariance can be obtained using a combination of a dark current image and a flat-field image, i.e., an image with all light blocked, and an image of a spectrally flat, evenly illuminated reflector. So, if

$$DN = mL + b + n$$

where: DN = recorded count (digital number),

m and b are gain and bias and n is noise.

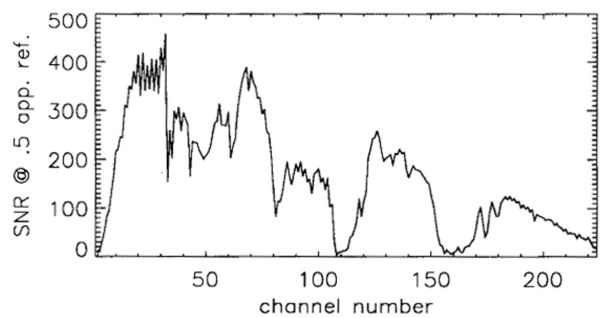
Then for L = 0, $DN_o = b + n$ is the dark current count. If the noise has zero mean, then the average dark current image is b. The gain is then given by:

$$m = \frac{DN - DN_o}{L_k} \quad \text{[counts per radiance unit]}$$

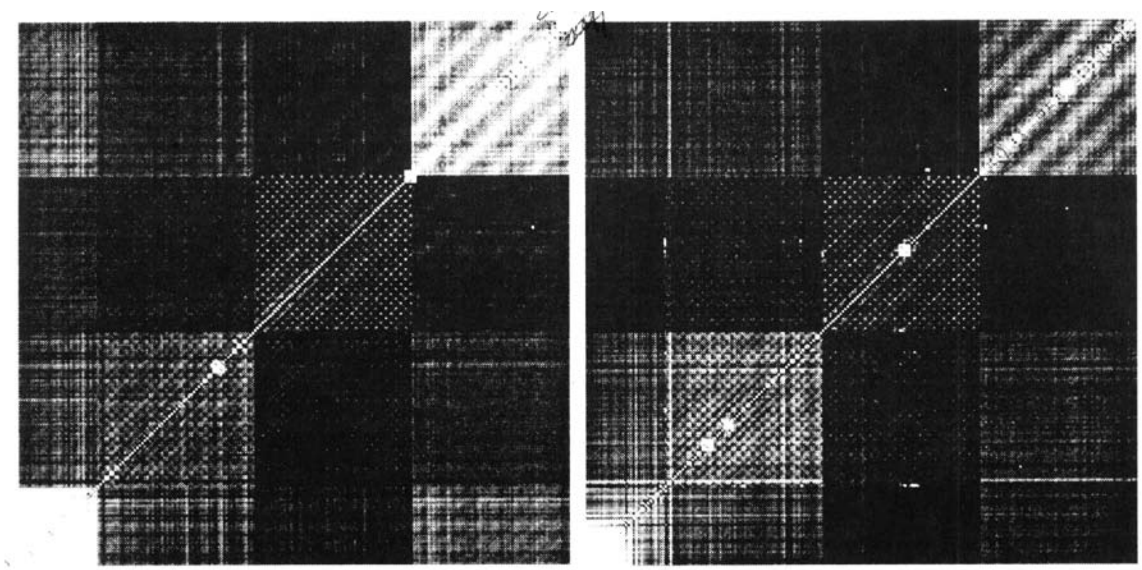
If n_{oi} is the standard deviation of the dark noise image in band i expressed in digital counts, then the standard deviation expressed in radiance units is: $\sigma_i = n_{oi}/m_i$ If $L_{i.5}$ is the radiance in the i^{th} band for a 50% reflector then the SNR for a 50% reflecting target is:

$$SNR = L_{i5}/\sigma_i$$

Comment: calibration involves recording DN_k at known radiance level (L_k) and DN_o dark (L=0).



Predicted S/N for a particular AVIRIS scene, based on a nominal 0.5 reflectance target.



Figures 3 and 4. Dark current covariance images for two different AVIRIS runs.

In-scene estimate of the noise covariance

An underlying assumption here is that the noise structure will be more accurately characterized by a mulit-dimensional normal distribution and will be uncorrelated with the information (signal) in the image data. A procedure to estimate the noise covariance, the Minimum/Maximum autocorrelation factors (MAF), has been designed[2]. The procedure exploits the fact that signal exhibit strong spatial correlation among nearby pixels in an image, while the spatial correlation for noise is very weak. The idea can be illustrated using two adjacent pixels, p_1 and p_2, with essentially the same target. Subtracting the two pixels then yields: $s_1 + n_1 - (s_2 + n_2) \approx n_1 - n_2$ where $s_1 \approx s_2$ is the signal and n_1 and n_2 are noise.

More precisely, let **B** be the transform that orthogonalizes the noise (i.e., **B** is the PCA transform for the noise image). If we transform the image data **Z**(x) using the noise eigenvectors, i.e., $\mathbf{Q}(x) = \mathbf{B}' \mathbf{Z}(x)$ and then normalize the $Q_i(x)$ values by $(u_{ni})^{1/2}$, then the noise variance in each of the normalized bands should be the same, i.e.,

$$R'(x) = \left(\frac{Q_1(x)}{\sqrt{u_1}}, \frac{Q_2(x)}{\sqrt{u_2}}, \cdots, \frac{Q_n(x)}{\sqrt{u_n}} \right)$$

where R(x) is the transformed image data (transformed using the eigenvectors of the noise covariance) normalized by the standard deviation of the noise in the transformed space. We have then "whitened" the noise in the transform space between the R_i bands.

We then compute the covariance matrix for the image data in the transform space and transform the R(x) data into a new principal component space. The PC's should now be ordered in a conventional fashion with the noise increasing with the PC rank.

The inherent dimensionality of the data is determined by examination of the final eigenvalues and the associated images. The data space can be divided into two parts: one part associated with large eigenvalues and coherent eigenimages, and a complementary part with near-unity eigenvalues and noise-dominated images. By using only the coherent portions, the noise is separated from the data, thus improving spectral processing results.

Noise removal and reconstruction of the original, noise-reduced image data

1. Eliminate the first several MNF Bands

This is roughly equivalent to eliminating the last several PC bands, except that the MNF bands have been adjusted to emphasize the noise content.

If the noise in the original images had been uncorrelated with the image data and the noise had been equal from band to band, the effect would be the same for the PCA and for the MNF processing.

2. Severely smooth the noise in the next several MNF bands.

For bands that appear to have some significant information but are still quite noisy, it can be useful to spatially smooth the data in order to minimize the noise while keeping the bulk of the information. The smoothing can be more severe than would be acceptable in the original image data since the noise has been largely isolated

3. Inverse Transform to obtain relatively noise-free image data.

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[2] Green, A.A. and P. Switzer (1984) Min/Max autocorrelation factors for multivariate spatial imagery. Tech. Report 6, Dept. of Statistics, Stanford University