On motor velocity control by using only position measurements: two case studies

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Abstract Velocity control by using only position measurements consists of finding a control algorithm that reconstructs the motor shaft velocity from the position measurements of the shaft encoder. The aim of this paper is to present the problem of velocity control in a linear description of a motor as paradigm in the application of the basic concepts of an introductory course in control systems; namely, transfer function, characteristic equation, stability, and Routh-Hurwitz criterion. Experiments have been carried out on a motor illustrating the performance of the velocity control algorithms discussed in this paper.

Keywords Routh-Hurwitz criterion; stability; velocity control; velocity estimation

In practice, the implementation of many control algorithms on motor drives requires velocity measurements. However, since the velocity measurement can be contaminated by noise, hence the control system performance may be reduced. The presence of noise in the measurements and the discretization of the controller also limits the values of the controller gains. Thanks to high-resolution encoders and hardware that allow a high sampling frequency, we could approach the velocity measurements via numerical differentiation to achieve control of motor drives. Notwithstanding, one generally finds that this approximation does not work well, especially as the sampling interval decreases, due to the encoder measurement noise.[1]

For this reason, the problem of finding control algorithms that deal in some way with the velocity reconstruction has been an active research topic. Interesting theoretical justification can be found in Ref. [2] (and references therein) for the solution in the case of robot motion control.

Recalling basic concepts from automatic control, this paper presents a theoretical and experimental comparison between two velocity control approaches to a linear description of a motor. The first one assumes velocity measurements, while the second one deals with velocity control by using only position measurements.

Based on the ideas discussed in Refs [2] and [3], the proposed solution to the problem of velocity control by using only position measurements follows the idea of replacing the motor shaft velocities $\dot{q}$ by a filtering of the motor shaft angular positions q and the desired velocity $\dot{q}_d$ via a stable first-order filter with zero relative degree.

A classical linear description of a direct current (d.c.) motor considering the torque as the input is given by[4,5]

$$J\ddot{q} + f_v\dot{q} = \tau, \tag{1}$$

where τ is the torque input, $J > 0$ is the rotor inertia and $f_v > 0$ is the viscous friction parameter.

Under the assumption that the motor parameters (J and f_v) are known, but motor shaft velocity $\dot{q}$ is unmeasurable, the control problem is to find a control law for the motor system (1) so that

$$\lim_{t\to\infty} \dot{\tilde{q}}(t) = \lim_{t\to\infty}\left[\dot{q}_d(t) - \dot{q}(t)\right] = 0,$$

where $\dot{q}_d(t)$ is the desired motor shaft velocity, which is assumed to be continuously differentiable.

Velocity control algorithms

This section describes two velocity control algorithms for the motor model (1). The first controller considers the use of velocity measurements and the second controller considers only position measurements. A comparison between these controllers from a structural point of view is presented.

Velocity control by using velocity measurements

An approach to velocity control is given by the following scheme based on the inverse dynamics technique[5]

$$\tau = J\left[\ddot{q}_d + k_v\dot{\tilde{q}} + k_i z\right] + f_v\dot{q}_d, \tag{2}$$

$$\dot{z} = \dot{\tilde{q}}, \tag{3}$$

where k_v and k_i are positive constants, and $\dot{\tilde{q}} = \dot{q}_d - \dot{q}$ denotes the velocity error. Figure 1 shows a block diagram of controller (2)–(3). Hereafter, the controller (2)–(3) will be referred to as VM (Velocity Measurements).

Substituting equation (2) in the motor model (1) we obtain the closed-loop dynamics

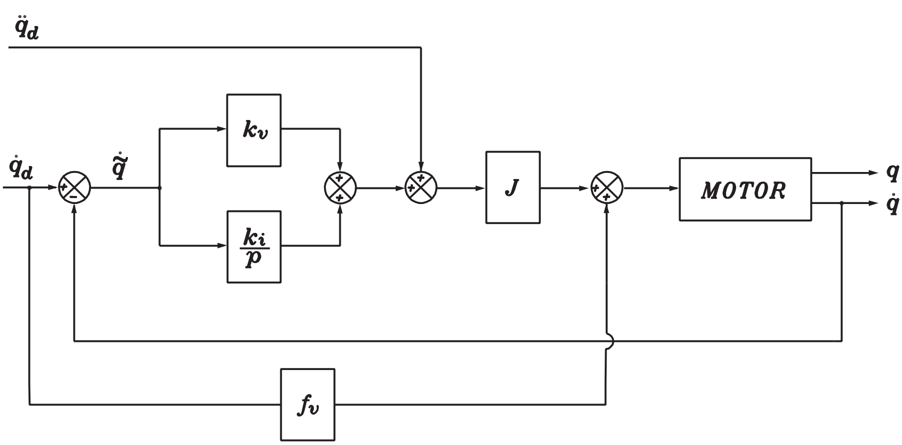

Fig. 1 *Velocity control by using velocity measurements.*

$$\ddot{q} + \left[k_v + \frac{f_v}{J}\right]\dot{\tilde{q}} + k_i z = 0, \tag{4}$$

where z is given by equation (3). The linear system (4) is stable since $k_v + \frac{f_v}{J} > 0$ and $k_i > 0$ as established from the Routh-Hurwitz criterion.[4,6]

The control law (2) considers desired friction compensation thanks to the term $f_v\dot{q}_d$. However, velocity control can also be attained using friction compensation:

$$\tau = J[\ddot{q}_d + k_v\dot{\tilde{q}} + k_i z] + f_v\dot{q},$$

leading also to a stable closed-loop system.

Velocity control by using only position measurements

Consider the controller given by

$$\tau = J\left[\ddot{q}_d + k_v\dot{\xi} + k_i\xi\right] + f_v\dot{q}_d, \tag{5}$$

where k_v and k_i are positive constants. As it can be noted, the proposed controller (5) has the same structure as controller (2), including friction feedforward $f_v\dot{q}_d$. The signal $\dot{\xi}$ in controller (5) is rendered as an estimation of the velocity error $\dot{\tilde{q}}$ used in controller (2).

Since now the velocity of the motor shaft is assumed unmeasurable, there should be an estimator of the velocity error $\dot{\tilde{q}}$. We propose the following first-order linear observer

$$\ddot{\xi} = -a\left[\dot{\xi} - \dot{\tilde{q}}\right], \tag{6}$$

where $a > 0$ is the observer gain. By introducing the variable

$$\vartheta = \dot{q}_d - \dot{\xi}, \tag{7}$$

and using the definition of $\dot{\tilde{q}} = \dot{q}_d - \dot{q}$, the observer (6) can be rewritten as

$$\dot{\vartheta} = -a\vartheta + a\dot{q} + \ddot{q}_d, \tag{8}$$

which is a first-order filter with zero relative degree and transfer function given by

$$\vartheta = \frac{p}{p+a}\left[aq + \dot{q}_d\right], \tag{9}$$

where $p = d/dt$ denotes the differential operator. From equation (7) we have that

$$\dot{\xi} = \dot{q}_d - \vartheta, \tag{10}$$

which is similar to the definition of the velocity error $\dot{\tilde{q}} = \dot{q}_d - \dot{q}$ in controller (2). Hence, ϑ can be considered as an estimation of the motor shaft velocity.

It is possible to demonstrate that system (9) is obtained from the following linear system which has as inputs the shaft position q and the desired shaft velocity $\dot{q}_d$:

$$\dot{x} = -a\vartheta, \tag{11}$$

$$\vartheta = x + aq + \dot{q}_d. \tag{12}$$

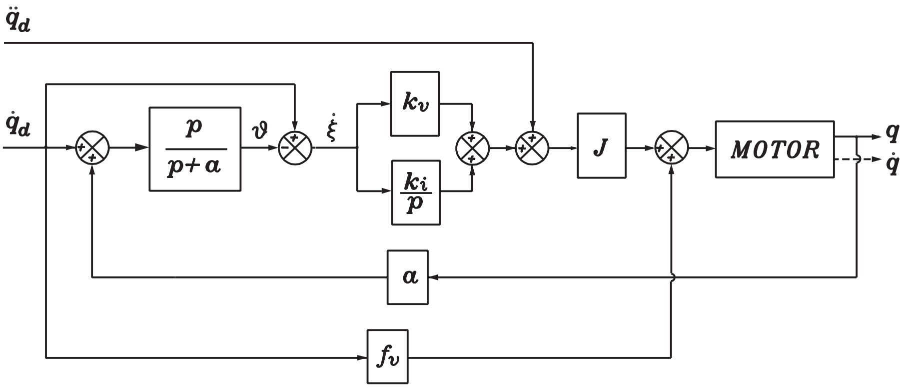

Fig. 2 *Velocity control by using only position measurements.*

In this way, controller (5) can be implemented using equation (10) and filter (11)–(12), as depicted in Fig. 2. The controller (5) will be referred to as OPM (Only Position Measurements).

Substituting (5) in motor equation (1) we have

$$\ddot{\tilde{q}} + \frac{f_v}{J}\dot{\tilde{q}} + k_v\dot{\xi} + k_i\xi = 0. \tag{13}$$

In conclusion, equations (13) and (6) represent the closed-loop dynamics which, expressed in state variables, is

$$\frac{d}{dt}\begin{bmatrix}\xi\\ \dot{\xi}\\ \dot{\tilde{q}}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & -a & a\\ -k_i & -k_v & -\frac{f_v}{J}\end{bmatrix}\begin{bmatrix}\xi\\ \dot{\xi}\\ \dot{\tilde{q}}\end{bmatrix}. \tag{14}$$

Equation (14) has the form $\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x}$, where $\mathbf{A} \in \Re^{3\times 3}$ and $\boldsymbol{x} \in \Re^{3}$ is the state vector. The matrix $\mathbf{A}$ is stable if the roots of the characteristic equation

$$\det\{sI - \mathbf{A}\} = 0$$

have a negative real part.[4,6] The characteristic equation of the system (14) is given by

$$s^3 + \left[a + \frac{f_v}{J}\right]s^2 + \left[ak_v + a\frac{f_v}{J}\right]s + ak_i = 0. \tag{15}$$

We can use the Routh-Hurwitz[4,6] criterion to find the conditions under which the characteristic equation (15) has roots with a negative real part. The necessary and sufficient condition for the characteristic equation (15) to have roots with a negative real part is that its coefficients be positive, i.e.

$$a + \frac{f_v}{J} > 0, \tag{16}$$

$$ak_v + a\frac{f_v}{J} > 0, \tag{17}$$

$$ak_i > 0, \tag{18}$$

and that

$$\left[a + \frac{f_v}{J}\right]\left[k_v + \frac{f_v}{J}\right] - k_i > 0, \tag{19}$$

be satisfied. With the knowledge of the motor parameters we can adequately tune the controller so that the closed-loop system is stable.

Let us consider a second alternative to the problem of velocity control using only position measurements:

$$\tau = J\left[\ddot{q}_d + k_v\dot{\xi} + k_i\xi\right] + f_v\vartheta, \tag{20}$$

$$\dot{\xi} = \dot{q}_d - \vartheta, \tag{21}$$

where ϑ is given by (12). Substituting (20) into (1) we have

$$\ddot{\tilde{q}} + \frac{f_v}{J}\dot{\tilde{q}} + \left[k_v - \frac{f_v}{J}\right]\dot{\xi} + k_i\xi = 0. \tag{22}$$

Thus, we can obtain conditions for the stability of the closed-loop system (6) and (22) using the Routh-Hurwitz criterion as before.

Practical implications

Most of the motor drives are provided with an optical encoder for the shaft displacement measurement. If the encoder resolution and the sampling frequency are high, the shaft velocities can be estimated by means of the Euler approximation

$$\dot{q}(ih) = \frac{q(ih) - q([i-1]h)}{h}, \tag{23}$$

where i is the discrete time and h is the sampling period.

However, it is interesting to review the practical implications of implementing the velocity controller (5) and (10), which use filter (11)–(12). By using the backward difference method, and considering (11)–(12), the practical implementation of the velocity controller (5) and (10) includes the velocity estimation as

$$\vartheta(ih) = \frac{1}{1+ah}\left[a\left[q(ih) - q([i-1]h)\right] + \vartheta([i-1]h) + h\ddot{q}_d(ih)\right]. \tag{24}$$

In this way, if $a = h^{-1}$, equation (24) becomes

$$\vartheta(ih) = \frac{1}{2}\left[\frac{q(ih) - q([i-1]h)}{h} + \vartheta([i-1]h)\right] + \frac{1}{2}h\ddot{q}_d(ih). \tag{25}$$

The velocity estimation given by (25) consists of the average between the actual estimated velocity via Euler approximation (23) and the former estimation, plus a compensation term given by $\frac{1}{2}h\ddot{q}_d(ih)$. In practice, by using (25) in lieu of (23) we would expect an improvement in the velocity estimation since the errors due to stochastic effects can be rejected by averaging.

Experimental results

Experiments on a direct-drive motor have been carried out in order to show the performance of the velocity controllers VM, given by (2) and (3), and OPM, given by (5) and (10). The motor used in the experiments is the model DM 1004C from Parker Compumotor which is operated in torque mode, so the motor acts as a torque source and accepts an analog voltage as reference torque signal. This motor is equipped with an encoder which provides a resolution of 655 360 pulses per revolution and can provide 4 Nm of maximum torque and 900 [deg/s] of maximum velocity. The control algorithm is written in C language and executed at 1 [ms] sampling rate on a PC with the data acquisition board MFIO-3A from Precision MicroDynamics.

Let us note that experiments have shown that static and Coulomb friction are present at the motor shaft. Thus, they act as disturbances of the closed-loop system. The numerical parameters of the motor are listed in Table 1.

Desired velocity trajectory and performance criterion

The performance of the controllers was tested by considering the desired velocity trajectory given by

$$\dot{q}_d(t) = 5.655t^2e^{-1.8t^3} + 11.781t^2e^{-1.8t^3}\sin(\omega t) + 2.1816\omega\left[1 - e^{-1.8t^3}\right]\cos(\omega t)\,[\text{rad/s}]. \quad (26)$$

It should be noted that at the beginning of the test (time $t = 0$ [s]) the desired velocity and acceleration are both zero.

The performance of the controllers was tested using $\omega = 6$ [rad/s] and $\omega = 2$ [rad/sec]. Figure 3 shows the time evolution of the desired velocity (26). It is easy to show that the desired velocity satisfies

$$|\dot{q}_d| \leq \begin{cases} 750.0\,[\text{deg/s}] \text{ for } \omega = 6\,[\text{rad/s}], \\ 281.2\,[\text{deg/s}] \text{ for } \omega = 2\,[\text{rad/s}], \end{cases}$$

thus, they are inside the allowed velocity band prescribed by the motor manufacturer.

TABLE 1 *Parameters of the motor*

	Notation	Value	Unit
Inertia rotor	J	2.5×10^{-3}	kg m^2/rad
Viscous friction	f_v	0.1438	Nm s/rad

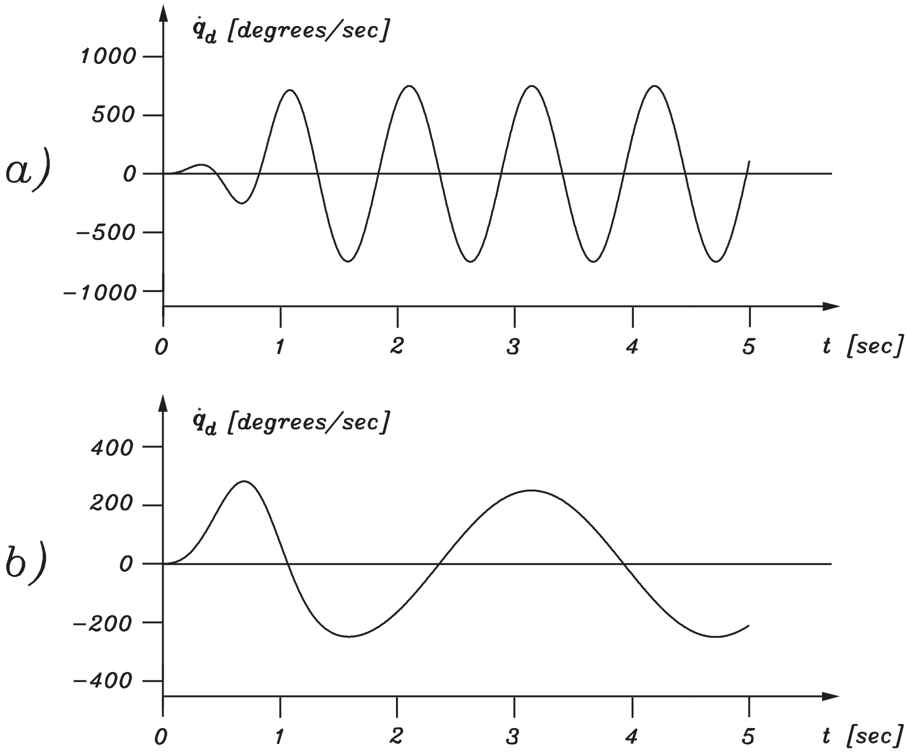

Fig. 3 *Time evolution of the desired velocity. (a) Using $\omega = 6$ [rad/s], (b) Using $\omega = 2$ [rad/s].*

The time evolution of the velocity error $\dot{\tilde{q}}$ reflects how well the control system performs. The performance criterion considered in this paper was the Root Mean Square (r.m.s.) value of the velocity error truncated Euclidean norm on a trip of time T, that is

$$L_T^2[\dot{\tilde{q}}] = \sqrt{\frac{1}{T}\int_0^T |\dot{\tilde{q}}(\sigma)|^2 \, d\sigma} \; [\text{rad/s}]. \tag{27}$$

The L_T^2 norm has been previously used by several authors as a criterion of tracking performance.[1,7,8] In practice, the discrete implementation of the criterion (27) leads to

$$L_T^2[\dot{\tilde{q}}] = \sqrt{\frac{1}{T}\sum_{k=0}^{i} |\dot{\tilde{q}}(kh)|^2 h} \; [\text{rad/s}].$$

where h = 1 [ms] is the sampling period and T = 5 [s] is the trip time.

Velocity control experiments

Experiments were carried out with controller VM, given by (2) and (3), and controller OPM, given by equations (5) and (10). The following gains were chosen

$$k_v = 200.0\,[1/\mathrm{s}],$$

$$k_i = 10000.0\,[1/\mathrm{s}^2],$$

Moreover, $a = 300.0$ [s^{-1}] was used in the filter (11)–(12) used in the OPM control algorithm.

As the sampling rate is relatively small (1 [ms]) and the encoder resolution high, the velocity estimation given by the Euler approximation (23) should be sufficient for consideration as a satisfactory velocity measurement. Thus, for the implementation of the controller VM in equations (2) and (3), the velocity error $\dot{\tilde{q}} = \dot{q}_d - q$ was computed with $\dot{q}$ given by the Euler approximation (23). For implementation of the controller OPM, given by (5) and (10), the velocity error $\dot{\tilde{q}}$ was computed in the same fashion as that in controller VM, through the Euler approximation (23), with the aim of comparing the performance of both controllers.

The velocity errors obtained with $\omega = 6$ [rad/s] are depicted in Fig. 4. The peaks observed in the velocity error time evolution are attributed to Coulomb friction, because they occur when the velocity of the motor shaft changes its sign. The VM controller performance shows maximum peaks of velocity error around 100 [deg/s], while the OPM controller performance shows peaks of 75 [deg/s], which means a 25% improvement in the latter compared to the former.

With $\omega = 2$ [rad/s] the velocity error showed a similar behavior in both controllers (see Fig. 5). The maximum peak of velocity error observed for the VM control algorithm was 50 [deg/s], while that for the OPM control algorithm was 42 [deg/s], a 16% improvement with respect to the VM control algorithm.

Discussion

Although global asymptotic stability implies that the velocity error $\dot{\tilde{q}}$ must vanish asymptotically, in practice Figs 4 and 5 reveal a steady state oscillatory behaviour. This is due to several factors such as uncompensated Coulomb friction and discrete controller implementation.

Visual examination of Figs 4 and 5 is not suitable for quantitative description of tracking errors, but the L_T^2 truncated norm criterion gives a useful performance index as shown in Table 2. In order to average out stochastic influences, Table 2 shows the mean of the r.m.s. value of the velocity error for ten runs.

For the two tested frequencies, the velocity controller OPM had the best performance compared with the VM, since the former had the bigger variation of the L_T^2 criterion. The OPM control had better repeatability and the average of L_T^2 for the ten runs was 22.30% and 11.16% less with respect to the VM control for $\omega = 6$ [rad/s], and $\omega = 2$ [rad/s], respectively.

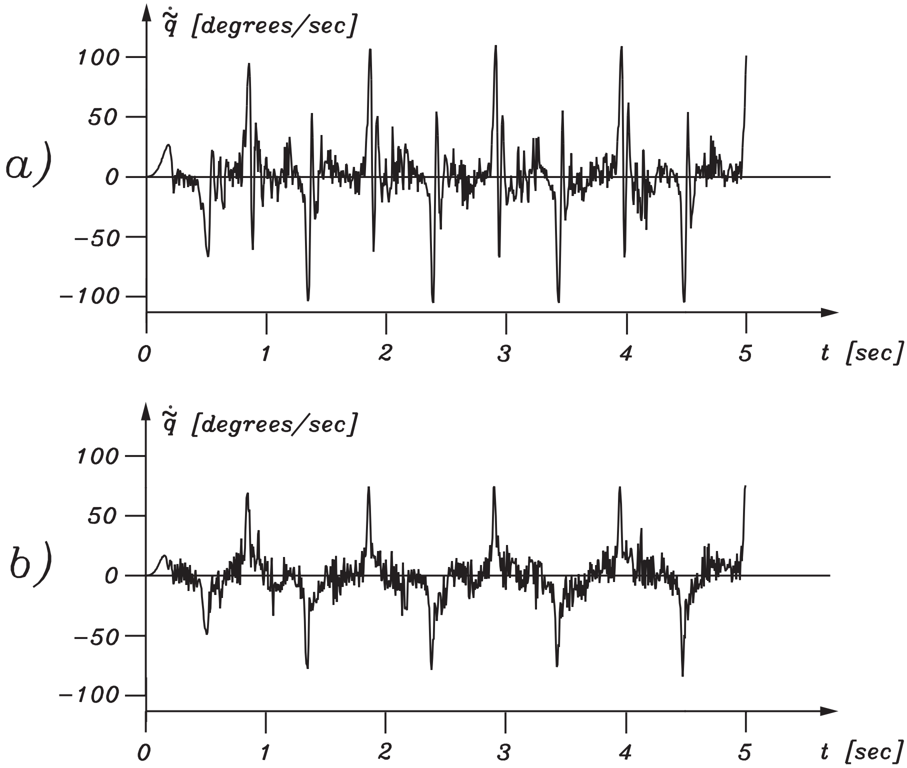

Fig. 4 *Time evolution of* $\dot{\tilde{q}}$ *using* $\omega = 6$ *[rad/s]. (a) VM controller, (b) OPM controller.*

TABLE 2 *Performance based on* $L_T^2[\dot{\tilde{q}}]$ *truncated norm [deg/s]*

ω	Algorithm	Average	Maximum	Minimum	Variation = max − min
6 [rad/s]	VM	27.00	27.48	26.55	0.93
	OPM	20.99	21.22	20.72	0.50
2 [rad/s]	VM	11.29	11.41	11.20	0.21
	OPM	10.03	10.07	09.99	0.08

Conclusion

Drawing basic concepts from an introductory course in automatic control, this paper studies the problem of motor velocity control by using only position measurements. It has been shown that with a simple replacing of the motor shaft velocity by a filtering of the motor shaft position and desired velocity it is possible to achieve velocity control. The simplicity of the proposed approach to velocity control as well as the experimental evidence illustrating improvement in performance compared with the traditional approach, makes it attractive for many applications.

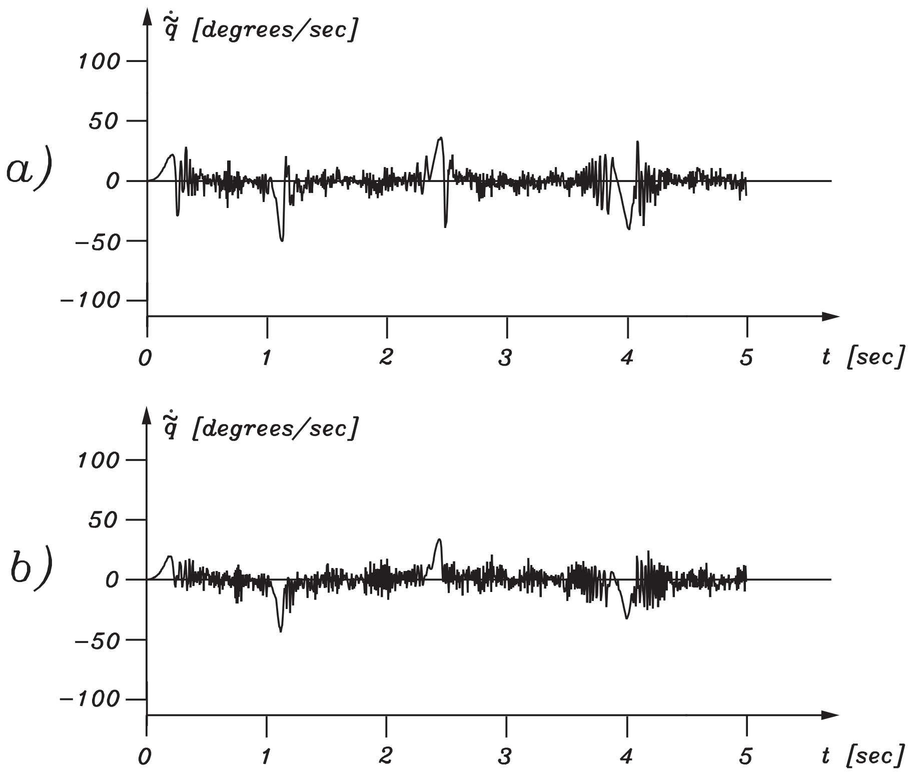

Fig. 5 *Time evolution of* $\dot{\tilde{q}}$ *using* $\omega = 2$ *[rad/s]. (a) VM controller, (b) OPM controller.*

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