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A Practical PID Regulator with Bounded Torques for Robot Manipulators

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Abstract: This paper proposes a saturated nonlinear PID regulator for industrial robot manipulators. Our controller considers the natural saturation problem given by the output of the control computer, the saturation phenomena of the internal PI velocity controller in the servo driver, and the actuator torque constraints of the robot manipulator. An approach based on the singular perturbations method is used to analyze the exponential stability of the closed-loop system. Experimental essays show the feasibility of the proposed controller. Furthermore, the theoretical results justify why the classical PID used in industrial robots preserves its exponential stability despite the saturation effects of the electronic control devices and the actuator torque constraints.

Keywords: Bounded torques, PID control, singular perturbations, stability analysis.

1. INTRODUCTION

It is well known that industrial robots are equipped with PID controllers which practically assure the semiglobal asymptotic stability of the closed-loop equilibrium for the regulation case [1-8]. Also, it is known that real-life actuators are unable to supply unlimited torque, and their output is bounded. This implies physical constraints which, if they are not taken into account in the design of the controller, can affect the stability and performance of the closed-loop system [9-14].

Furthermore, industrial robots are equipped with a position control computer which produces the commands of desired joint velocities. These commands are bounded in the same way that the actuator outputs are. In this work we consider these two constraints to design our controller.

Several approaches have been proposed in the literature in order to face up the regulation problem of robot manipulators with bounded inputs; different analytical frameworks and control objectives are considered [9-14]. Solutions without considering velocity mesurements with gravity compensation are treated in [12]. A full-state (position and velocity) feedback solution with adaptive gravity compensation is presented in [15]. More recently, new schemes dealing with the regulation problem of robot manipulators with bounded inputs have been presented: [16-20]. An adaptive approach involving task-space coordinates considering the uncertainties of the kinematic model of the robot manipulator is proposed in [18].

Some works that deal with global nonlinear PID regulators based on Lyapunov and passivity theory have been reported in [21-24], but without considering the influence of the saturation phenomena. Recently, a particular case of the class of nonlinear PID global regulators originally proposed in [23] was presented in [25].

A few saturated PID controllers (that is, bounded PID controllers taking into account the actuator torque constraints) have been reported: for the case of semiglobal asymptotic stability, a saturated linear PID controller was presented in [19] and [20]; for the case of global asymptotic stability, saturated nonlinear PID controllers were introduced in [26-28].

In this paper we propose a new saturated nonlinear PID regulator for robot manipulators that considers the torque constraints of the actuators. The structure of this proposed controller is closer to the structure of the practical PID controllers used in the industry. Fig. 1 shows the scheme that we consider to design our saturated nonlinear PID controller, where we clearly show the constraints over the input and output commands of the servo driver. We use a proportional controller as external position control and a joint velocity PI controller which is intrinsic to the servo drivers of each of the joints actuators in industrial robot manipulators.

We employ the singular perturbation theory to analyze exponential stability of the closed-loop system equilibrium. This result guarantees that exponential stability of the classical PID linear regulator in industrial robots is preserved although the saturation regions of the electronic devices and/or the actuators are reached.

The paper is organized as follows: Section 2 states the dynamic model of an n-link serial rigid robot manipula-

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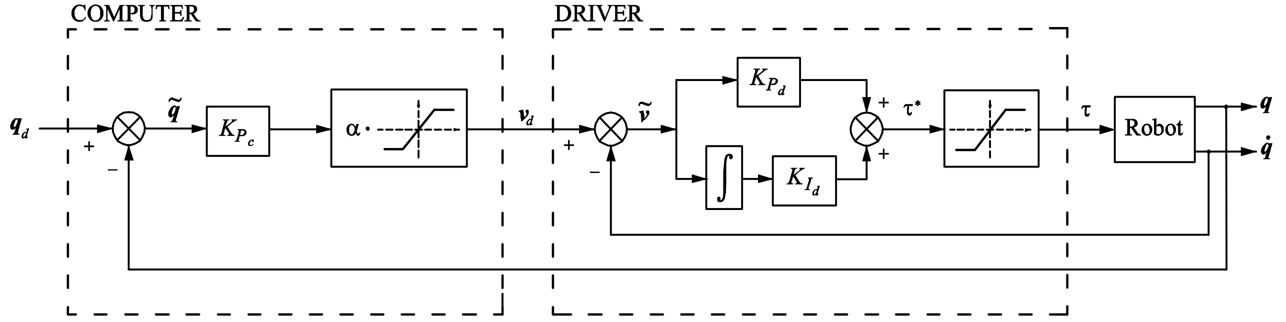

Fig. 1. Scheme of the practical nonlinear PID controller with bounded torques of robot manipulators.

tor in open-loop, some of its properties, as well as some considerations, assumptions and definitions that are useful throughout the analysis. The proposed control scheme is presented in Section 3. Section 4, shows the singular perturbed system to analyze. Section 5 states the stability analysis and proves that the control objective is achieved. An experimental essay is presented in Section 6. Finally, some conclusions are given in Section 7.

Throughout this paper we use the notation $\lambda_{\min}\{A(\mathbf{x})\}$ and $\lambda_{\max}\{A(\mathbf{x})\}$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(\mathbf{x})$, for any $\mathbf{x}\in\mathbb{R}^n$. Also, we define $\lambda_{\min}\{A\}$ as the greatest lower bound (infimum) of $\lambda_{\min}\{A(\mathbf{x})\}$, for all $\mathbf{x}\in\mathbb{R}^n$, that is, $\lambda_{\min}\{A\}=\inf_{\mathbf{x}\in\mathbb{R}^n}\lambda_{\min}\{A(\mathbf{x})\}$. Similarly, we define $\lambda_{\max}\{A\}$ as the least upper bound (supremum) of $\lambda_{\max}\{A(\mathbf{x})\}$, for all $\mathbf{x}\in\mathbb{R}^n$, that is, $\lambda_{\max}\{A\}=\sup_{\mathbf{x}\in\mathbb{R}^n}\lambda_{\max}\{A(\mathbf{x})\}$. Vectors are denoted by bold lowercase letters. The norm of vector $\mathbf{x}$ is defined as $\|\mathbf{x}\|=\sqrt{\mathbf{x}^T\mathbf{x}}$ and that of matrix $A(\mathbf{x})$ is defined as the induced norm $\|A(\mathbf{x})\|=\sqrt{\lambda_{\max}\{A(\mathbf{x})^T A(\mathbf{x})\}}$.

2. PRELIMINARIES

2.1. Robot dynamics

The dynamics of an n-link serial rigid robot, without the effect of friction, can be written as [29]:

$$M(\mathbf{q})\ddot{\mathbf{q}}+C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})=\boldsymbol{\tau}, \tag{1}$$

where $\mathbf{q}\in\mathbb{R}^n$ is the vector of joint positions, $\dot{\mathbf{q}}\in\mathbb{R}^n$ is the vector of joint velocities, $\boldsymbol{\tau}\in\mathbb{R}^n$ is the vector of applied torques, $M(\mathbf{q})\in\mathbb{R}^{n\times n}$ is the symmetric positive definite manipulator inertia matrix, $C(\mathbf{q},\dot{\mathbf{q}})\in\mathbb{R}^{n\times n}$ is the matrix of centripetal and Coriolis torques and $\mathbf{g}(\mathbf{q})\in\mathbb{R}^n$ is the vector of gravitational torques, obtained as the gradient of the robot potential energy $\mathcal{U}(\mathbf{q})$, i.e.,

$$\mathbf{g}(\mathbf{q})=\frac{\partial\mathcal{U}(\mathbf{q})}{\partial\mathbf{q}}. \tag{2}$$

We assume that all the joints of the robot are of revolute type.

2.2. Properties of the robot dynamics

We recall two important properties of the robot dynamics (1) which are useful in our paper:

Property 1: The matrix $C(\mathbf{q},\dot{\mathbf{q}})$ and the time derivative $\dot{M}(\mathbf{q})$ of the inertia matrix satisfy [30,31]:

$$\dot{\mathbf{q}}^T\left[\frac{1}{2}\dot{M}(\mathbf{q})-C(\mathbf{q},\dot{\mathbf{q}})\right]\dot{\mathbf{q}}=\mathbf{0},\qquad \forall\ \mathbf{q},\dot{\mathbf{q}}\in\mathbb{R}^n.$$

Property 2: The gravitational torque vector $\mathbf{g}(\mathbf{q})$ is bounded for all $\mathbf{q}\in\mathbb{R}^n$. This means that there exist finite constants $\gamma_i\ge 0$ such that [32]:

$$\sup_{\mathbf{q}\in\mathbb{R}^n}|g_i(\mathbf{q})|\le\gamma_i,\qquad i=1,2,\cdots,n, \tag{3}$$

where $g_i(\mathbf{q})$ stands for the i-th element of $\mathbf{g}(\mathbf{q})$. Equivalently, there exists a constant k' such that

$$\|\mathbf{g}(\mathbf{q})\|\le k', \quad \text{for all } \mathbf{q}\in\mathbb{R}^n.$$

Furthermore, there exists a positive constant k_g such that

$$\left\|\frac{\partial\mathbf{g}(\mathbf{q})}{\partial\mathbf{q}}\right\|\le k_g,$$

for all $\mathbf{q}\in\mathbb{R}^n$, and

$$\|\mathbf{g}(\mathbf{x})-\mathbf{g}(\mathbf{y})\|\le k_g\,\|\mathbf{x}-\mathbf{y}\|,$$

for all $\mathbf{x},\mathbf{y}\in\mathbb{R}^n$.

2.3 Problem formulation

Before presenting the formulation of the control problem, we recall some useful definitions.

Definition 1: The hard saturation function is denoted by $\mathbf{sat}(\mathbf{x};\mathbf{k}) \in \mathbb{R}^n$, where

$$\mathbf{sat}(\mathbf{x};\mathbf{k}) = \begin{bmatrix} \mathrm{sat}(x_1;k_1) \\ \mathrm{sat}(x_2;k_2) \\ \vdots \\ \mathrm{sat}(x_n;k_n) \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{k} = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}$$

with k_i being the i-th saturation limit, $i = 1,2,\ldots,n$, and each element of $\mathbf{sat}(\mathbf{x};\mathbf{k})$ being defined as:

$$\mathrm{sat}(x_i;k_i) = \begin{cases} x_i, & \text{if } |x_i| \le k_i, \\ k_i, & \text{if } x_i > k_i, \\ -k_i, & \text{if } x_i < -k_i. \end{cases}$$

The control scheme proposed in this paper involves special saturation functions that fit in the following definition.

Definition 2 [16]**:** Given positive constants l and m, with $l < m$, a function $\mathrm{Sat}(x;l,m): \mathbb{R} \to \mathbb{R} : x \mapsto \mathrm{Sat}(x;l,m)$ is said to be a strictly increasing linear saturation function for (l,m) if it is locally Lipschitz, strictly increasing, C^2 differentiable, and satisfies:

1) $\mathrm{Sat}(x;l,m) = x$ when $|x| \le l$.

2) $|\mathrm{Sat}(x;l,m)| < m$ for all $x \in \mathbb{R}$.

For instance, the following saturation function is a special case of a linear saturation given in Definition 2:

$$\mathrm{Sat}(x;l,m) = \begin{cases} -l + (m-l)\tanh\left(\dfrac{x+l}{m-l}\right), & \text{if } x < -l, \\ x, & \text{if } |x| \le l, \\ l + (m-l)\tanh\left(\dfrac{x-l}{m-l}\right), & \text{if } x > l. \end{cases} \tag{4}$$

The hard saturation function does not belong to Definition 2 because it is not C^2 differentiable and $l = m$.

n-saturation functions can be joined together in an $n \times 1$ saturation function vector denoted by $\mathbf{Sat}(\mathbf{x};\mathbf{l},\mathbf{m})$, i.e.,

$$\mathbf{Sat}(\mathbf{x};\mathbf{l},\mathbf{m}) = \begin{bmatrix} \mathrm{Sat}(x_1;l_1,m_1) \\ \mathrm{Sat}(x_2;l_2,m_2) \\ \vdots \\ \mathrm{Sat}(x_n;l_n,m_n) \end{bmatrix},$$

where $\mathbf{x},\mathbf{l},\mathbf{m} \in \mathbb{R}^n$, that is,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{l} = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{bmatrix}, \quad \mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix}.$$

Consider the robot dynamics (1). Assume that each joint actuator is able to supply a known maximum torque $\tau_i^{\max}$ so that:

$$|\tau_i| \le \tau_i^{\max}, \qquad i = 1,2,\ldots,n, \tag{5}$$

where τ_i stands for the i-th entry of vector $\boldsymbol{\tau}$. In other words, if u_i represents the control signal (controller output) before the actuator, related to the i-th joint, then

$$\tau_i = \tau_i^{\max}\mathrm{sat}\left(\frac{u_i}{\tau_i^{\max}}\right) \tag{6}$$

for $i = 1,2,\ldots,n$, where $\mathrm{sat}(\cdot)$ is the hard saturation function with unitary limit. We also assume:

Assumption 1: The maximum torque $\tau_i^{\max}$ of each actuator satisfies the following condition:

$$\tau_i^{\max} > \gamma_i, \tag{7}$$

where γ_i was defined in Property 2, with $i = 1,2\ldots,n$. This assumption means that the robot actuators are able to supply torques in order to hold the robot at rest for all desired joint position $\mathbf{q}_d \in \mathbb{R}^n$.

The control problem is to design a controller to compute the torque $\boldsymbol{\tau} \in \mathbb{R}^n$ applied to the joints, satisfying constraints (5), such that the robot joint positions $\mathbf{q}$ tend asymptotically toward the constant desired joint positions $\mathbf{q}_d$.

3. PROPOSED CONTROL SCHEME

In this section we present a nonlinear PID controller which can be seen as a practical version of the classical PID control of robot manipulators.

As shown in Fig. 1, the proposed controller is formed by two loops: an outer joint-position proportional (P) loop and an inner joint-velocity proportional-integral (PI) loop. Without considering the saturation effects, in [33] it was proven that an outer-loop position P controller ($\mathbf{v}_d = K_{p_c}\tilde{\mathbf{q}}$ in Fig. 1) together with an inner velocity PI controller ($\boldsymbol{\tau} = K_{p_d}\tilde{\mathbf{v}} + K_{i_d}\int_0^t \tilde{\mathbf{v}}(r)dr$ in Fig. 1) conform a classical PID controller, that is:

$$\boldsymbol{\tau} = K_p\tilde{\mathbf{q}} + K_i\mathbf{w} - K_v\dot{\mathbf{q}},$$

$$\mathbf{w} = \int_0^t \tilde{\mathbf{q}}(r)dr,$$

where $K_p = K_{p_d}K_{p_c} + K_{i_d}$, $K_v = K_{p_d}$, $K_i = K_{i_d}K_{p_c} \in \mathbb{R}^{n \times n}$ are diagonal positive definite matrices, $\tilde{\mathbf{q}} = \mathbf{q}_d - \mathbf{q} \in \mathbb{R}^n$ is the position error vector, and $\tilde{\mathbf{v}} = \mathbf{v}_d - \dot{\mathbf{q}} \in \mathbb{R}^n$ is the velocity error vector.

However, in real applications the commands supplied by the computer (position P loop) are limited by intrinsic constraints of the electronic devices, and the servo-drivers (velocity PI loop) and the actuators are unable to supply unlimited torque commands and torques, respectively; hence we must take into account these constraints in the closed-loop stability analysis. Because the cascade connection of two saturation blocks can be shown by only one saturation function, and for simplicity, in Fig. 1 the saturation of the velocity PI loop and the saturation of the actuators, are both represented by the last saturation block; also we are assuming to have ideal torque controllers in the driver.

We consider that, according to Fig. 1, the position controller implemented by the computer is given by

$$\mathbf{v}_d = \alpha\,\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}};\mathbf{l}_p,\mathbf{m}_p), \tag{8}$$

where $K_{p_c} \in \mathbb{R}^{n\times n}$ is a diagonal positive definite matrix whose elements are $k_{p_{ci}}$ with $i = 1,2,\dots,n$, $\alpha > 0$ is a small suitably selected constant and $\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}};\mathbf{l}_p,\mathbf{m}_p)$ is a saturation function as in Definition 2, for some $(\mathbf{l}_p,\mathbf{m}_p)$, where $\mathbf{l}_p$ and $\mathbf{m}_p$ are vectors whose elements are l_{p_i} and m_{p_i}, respectively, with $i = 1,2,\dots,n$.

The joint-velocity PI controller, in practice, is naturally implemented into the servo-drivers as:

$$\boldsymbol{\tau}^* = K_{p_d}\tilde{\mathbf{v}} + K_{i_d}\int_0^t \tilde{\mathbf{v}}(r)\,dr, \tag{9}$$

where K_{p_d}, $K_{i_d} \in \mathbb{R}^{n\times n}$ are diagonal positive definite matrices, and $\tilde{\mathbf{v}}$ is the joint velocity error vector given by:

$$\begin{aligned}\tilde{\mathbf{v}} &= \mathbf{v}_d - \dot{\mathbf{q}}\\ &= \alpha\,\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}};\mathbf{l}_p,\mathbf{m}_p) - \dot{\mathbf{q}},\end{aligned} \tag{10}$$

Substituting (10) in (9) we obtain

$$\begin{aligned}\boldsymbol{\tau}^* &= \alpha K_{p_d}\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}};\mathbf{l}_p,\mathbf{m}_p) - K_{p_d}\dot{\mathbf{q}}\\ &\quad + K_{i_d}\int_0^t[\alpha\,\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}}(r);\mathbf{l}_p,\mathbf{m}_p) - \dot{\mathbf{q}}(r)]dr,\end{aligned}$$

which has the form of the nonlinear PID global regulator in [21,23], that is,

$$\begin{aligned}\boldsymbol{\tau}^* &= K_p\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}};\mathbf{l}_p,\mathbf{m}_p) - K_v\dot{\mathbf{q}}\\ &\quad + K_i\int_0^t[\alpha\,\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}}(r);\mathbf{l}_p,\mathbf{m}_p) - \dot{\mathbf{q}}(r)]dr\end{aligned}$$

with

$$K_p = \alpha K_{p_d},\ K_v = K_{p_d},\ K_i = K_{i_d},$$

where K_p, K_v, $K_i \in \mathbb{R}^{n\times n}$ are diagonal positive definite matrices whose elements are k_{p_i}, k_{v_i}, k_{i_i}, respectively, with $i = 1,2,\dots,n$.

Finally, due to the servo-drivers and the actuators are physically limited, the nonlinear PID controller naturally results in a nonlinear PID controller with bounded torques given by

$$\begin{aligned}\boldsymbol{\tau} &= \mathbf{Sat}\Big[K_p\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}};\mathbf{l}_p,\mathbf{m}_p) - K_v\dot{\mathbf{q}} + \mathbf{w};\mathbf{l}_{pi},\mathbf{m}_{pi}\Big],\\ \mathbf{w} &= K_i\int_0^t[\alpha\,\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}}(r);\mathbf{l}_p,\mathbf{m}_p) - \dot{\mathbf{q}}(r)]dr,\end{aligned} \tag{11}$$

where $\mathbf{Sat}\Big[K_p\mathbf{Sat}\Big(K_{p_c}\tilde{\mathbf{q}};\mathbf{l}_p,\mathbf{m}_p\Big) - K_v\dot{\mathbf{q}} + \mathbf{w};\mathbf{l}_{pi},\mathbf{m}_{pi}\Big]$ is a vector whose elements are strictly increasing linear saturation functions, such as those in Definition 2, for some $(\mathbf{l}_{pi},\mathbf{m}_{pi})$, where $\mathbf{l}_{pi}$ and $\mathbf{m}_{pi}$ are vectors whose elements are l_{pi_i} and m_{pi_i}, respectively, with $i = 1,2,\dots,n$, satisfying the following assumption.

Assumption 2: The saturation limits of the P and the PI loops satisfy:

$$\gamma_i < l_{pi_i} < m_{pi_i} < \tau_i^{\max}. \tag{12}$$

Remark 1: In practice, the saturation constraints of the electronic devices and the actuators are in fact hard saturations like those in Definition 1. However, with the end of carrying out the stability analysis, they can be approximated by saturation functions like those defined in Definition 2 with $l < m$ and l arbitrarily close to m.

In order to simplify the notation, henceforth, we will omit the limits in the argument of the saturation functions.

4. SINGULARLY PERTURBED SYSTEM

4.1. Closed-loop system

By substituting (11) into the robot dynamics (1), we obtain

$$\frac{d}{dt}\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\\ \mathbf{w}\end{bmatrix} = \begin{bmatrix}-\dot{\mathbf{q}}\\ M(\mathbf{q})^{-1}\Big[\mathbf{Sat}\Big[K_p\mathbf{Sat}\Big(K_{p_c}\tilde{\mathbf{q}}\Big) - K_v\dot{\mathbf{q}} + \mathbf{w}\Big] - C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{g}(\mathbf{q})\Big]\\ K_i\Big[\alpha\mathbf{Sat}\Big(K_{p_c}\tilde{\mathbf{q}}\Big) - \dot{\mathbf{q}}\Big]\end{bmatrix}, \tag{13}$$

which is an autonomous differential equation with a unique equilibrium point given by $\begin{bmatrix}\tilde{\mathbf{q}}^T & \dot{\mathbf{q}}^T & \mathbf{w}^T\end{bmatrix}^T = \begin{bmatrix}\mathbf{0}^T & \mathbf{0}^T & \mathbf{g}(\mathbf{q}_d)^T\end{bmatrix}^T \in \mathbb{R}^{3n}$, where we have used Assumption 2, that is, $l_{pi_i} > \gamma_i$ to get that $\mathbf{Sat}(\mathbf{w}) - \mathbf{g}(\mathbf{q}_d) = \mathbf{0} \Leftrightarrow \mathbf{w} = \mathbf{g}(\mathbf{q}_d)$. In order to move the equilibrium point of (13) to the origin, we apply the following change of variables $\mathbf{x} = \mathbf{w} - \mathbf{g}(\mathbf{q}_d)$. Now the new closed-loop system is given by:

$$\frac{d}{dt}\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\\ \mathbf{x}\end{bmatrix}=\begin{bmatrix}-\dot{\mathbf{q}}\\ M(\mathbf{q})^{-1}\Big[\mathbf{Sat}\Big[K_p\mathbf{Sat}\big(K_{P_c}\tilde{\mathbf{q}}\big)-K_v\dot{\mathbf{q}}\\ +\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\big]-C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}-\mathbf{g}(\mathbf{q})\big]\\ K_i\Big[\alpha\mathbf{Sat}\big(K_{P_c}\tilde{\mathbf{q}}\big)-\dot{\mathbf{q}}\Big]\end{bmatrix}. \quad (14)$$

The previous closed-loop system can be studied as a singularly perturbed system. To this end, the system (14) can be described as a two first-order differential equations as follows:

$$\frac{d}{dt}\mathbf{x} = K_i\Big[\alpha\,\mathbf{Sat}\big(K_{P_c}\tilde{\mathbf{q}}\big)-\dot{\mathbf{q}}\Big], \quad (15)$$

$$\frac{d}{dt}\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\end{bmatrix}=\begin{bmatrix}-\dot{\mathbf{q}}\\ M(\mathbf{q})^{-1}\Big[\mathbf{Sat}\Big[K_p\mathbf{Sat}\big(K_{P_c}\tilde{\mathbf{q}}\big)-K_v\dot{\mathbf{q}}\\ +\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\big]-C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}-\mathbf{g}(\mathbf{q})\big]\end{bmatrix}. \quad (16)$$

Moreover, by choosing the integral gain matrix as $K_i = \varepsilon K_i^*$, where K_i^* is a diagonal positive definite matrix and $\varepsilon > 0$ is a small parameter, and letting $t' = \varepsilon t$ be a new time-scale (t' is a slow time compared to t), we can rewrite (15)-(16) as

$$\frac{d}{dt'}\mathbf{x} = K_i^*\Big[\alpha\mathbf{Sat}\big(K_{P_c}\tilde{\mathbf{q}}\big)-\dot{\mathbf{q}}\Big], \quad (17)$$

$$\varepsilon\frac{d}{dt'}\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\end{bmatrix}=\begin{bmatrix}-\dot{\mathbf{q}}\\ M(\mathbf{q})^{-1}\Big[\mathbf{Sat}\Big[K_p\mathbf{Sat}\big(K_{P_c}\tilde{\mathbf{q}}\big)-K_v\dot{\mathbf{q}}\\ +\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\big]-C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}-\mathbf{g}(\mathbf{q})\big]\end{bmatrix}, \quad (18)$$

where, in the forthcoming analysis and in accordance with singular perturbations theory, $\mathbf{x}$ will be treated as a fixed parameter, due to its slow variation. The physical meaning of this singular perturbation structure that makes possible to regard $\mathbf{x}$ as a frozen variable can be described as follows [20]. For a short time, the saturated PD control actions (18) dominates over the integral control action (17). This PD action drives quickly the system trajectories into a neighborhood of the equilibrium point. Once into such neghborhood, i.e., when time is long, the integral action (seen as the slow subsystem (17)) acts to move the trajectory into the desired position $\mathbf{q}_d$.

4.2. Equilibrium points of (18)

For each fixed $\mathbf{x}$ representing the frozen variable as a fixed parameter in (18), the equilibrium points are the solutions of the nonlinear system:

$$\begin{aligned}&\dot{\mathbf{q}} = \mathbf{0},\\ &\mathbf{Sat}\Big[K_p\mathbf{Sat}\big(K_{P_c}\tilde{\mathbf{q}}\big)+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\Big]-\mathbf{g}(\mathbf{q})=\mathbf{0}.\end{aligned} \quad (19)$$

According to Definition 2 and Assumption 2 $(l_{pi_i} > \gamma_i)$, (19) works in the linear region of the external saturation, and then (19) can be written as:

$$K_p\mathbf{Sat}(K_{P_c}\tilde{\mathbf{q}})+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)-\mathbf{g}(\mathbf{q}_d-\tilde{\mathbf{q}}) = \mathbf{0}\in\mathbb{R}^n. \quad (20)$$

Under the following assumption we can establish sufficient conditions to assure a unique solution of (20). Suppose that k_{p_i} is chosen sufficiently large such that:

$$\begin{aligned}&k_{p_i}l_{p_i} > |x_i+g_i(\mathbf{q}_d)-g_i(\mathbf{q}_d-\tilde{\mathbf{q}})|,\\ &\forall\,\tilde{q}_i\in\mathbb{R},\ i=1,2,\ldots,n,\end{aligned} \quad (21)$$

then (20) is equivalent to:

$$K_pK_{P_c}\tilde{\mathbf{q}}+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)-\mathbf{g}(\mathbf{q}_d-\tilde{\mathbf{q}}) = \mathbf{0}\in\mathbb{R}^n. \quad (22)$$

Now, the Contraction Mapping Theorem [7,34] guarantees that (22) has a unique solution $\tilde{\mathbf{q}}=\mathbf{h}_1(\mathbf{x}) \in\mathbb{R}^n$ provided that:

$$\lambda_{\min}\{K_pK_{P_c}\} > k_g \quad (23)$$

is satisfied, where constant k_g is defined in Property 2.

Then, we have that, for each $\mathbf{x}\in\mathbb{R}^n$, the unique equilibrium point of (18) is:

$$\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\end{bmatrix}=\begin{bmatrix}\mathbf{h}_1(\mathbf{x})\\ \mathbf{0}\end{bmatrix}:=\mathbf{h}(\mathbf{x})\in\mathbb{R}^{2n}. \quad (24)$$

Consecuently, we have that:

$$\begin{aligned}\mathbf{x} &= \mathbf{h}_1^{-1}(\tilde{\mathbf{q}})\\ &= -K_p\mathbf{Sat}\big(K_{P_c}\tilde{\mathbf{q}}\big)-\mathbf{g}(\mathbf{q}_d)+\mathbf{g}(\mathbf{q}),\end{aligned} \quad (25)$$

which we will use later on.

4.3. Overall singularly perturbed system

In order to proceed with the stability analysis, we shift the equilibrium point of (18) to the origin. To this end, we make the following change of variables:

$$\begin{bmatrix}\mathbf{y}_1(t')\\ \mathbf{y}_2(t')\end{bmatrix}=\begin{bmatrix}\tilde{\mathbf{q}}(t')-\mathbf{h}_1(\mathbf{x})\\ \dot{\mathbf{q}}(t')\end{bmatrix}, \quad (26)$$

which implies that $\tilde{\mathbf{q}}=\mathbf{y}_1+\mathbf{h}_1(\mathbf{x})$. Then, (17)-(18) can be now represented by the new variables as a singular perturbed system given by

$$\frac{d}{dt'}\mathbf{x} = K_i^*\Big[\alpha\,\mathbf{Sat}\big(K_{P_c}(\mathbf{y}_1+\mathbf{h}_1(\mathbf{x}))\big)-\mathbf{y}_2\Big], \quad (27)$$

$$\varepsilon\frac{d}{dt'}\left[\begin{array}{c}\mathbf{y}_1\\ \hline \mathbf{y}_2\end{array}\right]=\left[\begin{array}{c}-\mathbf{y}_2\\ -\varepsilon\left[\dfrac{\partial\mathbf{h}_1(\mathbf{x})}{\partial\mathbf{x}}\right]K_i^*\left[\alpha\mathbf{Sat}\left(K_{p_c}(\mathbf{y}_1+\mathbf{h}_1(\mathbf{x}))\right)-\mathbf{y}_2\right]\\ \hline M(\mathbf{q}_d-\mathbf{y}-\mathbf{h}_1(\mathbf{x}))^{-1}\\ \times\left[\mathbf{Sat}\left[K_p\mathbf{Sat}\left(K_{p_c}(\mathbf{y}_1+\mathbf{h}_1(\mathbf{x}))\right)-K_v\mathbf{y}_2\right.\right.\\ \left.+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\right]\\ \left.-C(\mathbf{q}_d-\mathbf{y}_1-\mathbf{h}_1(\mathbf{x}),\mathbf{y}_2)\mathbf{y}_2-\mathbf{g}(\mathbf{q}_d-\mathbf{y}-\mathbf{h}_1(\mathbf{x}))\right]\end{array}\right]. \tag{28}$$

5. STABILITY ANALYSIS

According to the theory of singularly perturbed systems [34], the origin of (17)-(18) is asymptotically stable if and only if the origin of (27)-(28) is asymptotically stable. It is important to remember that $\mathbf{x}$ is a fixed parameter in (18) and (28), this is because t' and $\mathbf{x}$ are slowly varying since, in the t time scale, they are given by [34]:

$$t'=t_0+\varepsilon t,\qquad \mathbf{x}=\mathbf{x}(t_0+\varepsilon t,\varepsilon), \tag{29}$$

being t_0 the initial time. The setting of $\varepsilon=0$ freezes these variables at $t'=t_0$ and $\mathbf{x}=\mathbf{x}(t_0)$ (initial conditions).

By simplicity, we divide the stability analysis in two parts:

- First, we will prove asymptotic stability and local exponential stability of the origin of a saturated PD controller with desired gravity compensation plus a constant vector $\mathbf{x}$, which can be seen as a constant control input.
- Second, based on a theorem of singularly perturbed systems, we will prove that the origin of (17)-(18) is locally exponentially stable.

5.1. Stability analysis of a saturated PD controller with desired gravity compensation plus a constant vector x

The control law that describes the proposed saturated PD controller with desired gravity compensation plus a constant vector $\mathbf{x}$ is given by:

$$\boldsymbol{\tau}=\mathbf{Sat}\left[K_p\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}})-K_v\dot{\mathbf{q}}+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\right]. \tag{30}$$

By substituting (30) into the robot dynamics (1), we obtain

$$\frac{d}{dt}\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\end{bmatrix}=\begin{bmatrix}-\dot{\mathbf{q}}\\ M(\mathbf{q})^{-1}\left[\mathbf{Sat}\left[K_p\mathbf{Sat}\left(K_{p_c}\tilde{\mathbf{q}}\right)-K_v\dot{\mathbf{q}}\right.\right.\\ \left.\left.+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\right]-C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}-\mathbf{g}(\mathbf{q})\right]\end{bmatrix}, \tag{31}$$

The equilibrium points include the solutions of the nonlinear equation (19) which has already been proven to have a unique solution $\mathbf{h}_1(\mathbf{x})$; thus, the equilibrium is $[\tilde{\mathbf{q}}^T\ \dot{\mathbf{q}}^T]^T=[\mathbf{h}_1(\mathbf{x})^T\ \mathbf{0}^T]^T$, provided that $\lambda_{\min}\{K_pK_{p_c}\}>k_g$ is satisfied.

5.1.1 Asymptotic stability analysis

To carry out the stability analysis of the equilibrium of (31), we propose the following Lyapunov function candidate, which is inspired from [20]:

$$\begin{aligned}W(\tilde{\mathbf{q}},\dot{\mathbf{q}})=&\frac{1}{2}\dot{\mathbf{q}}^TM(\mathbf{q})\dot{\mathbf{q}}\\ &+\sum_{i=1}^{n}\int_0^{\tilde{q}_i}\mathrm{Sat}\left[k_{p_i}\mathrm{Sat}(k_{p_{c_i}}r_i)+x_i+g_i(\mathbf{q}_d)\right]dr_i\\ &+\mathscr{U}(\mathbf{q}_d-\tilde{\mathbf{q}})\\ &-\sum_{i=1}^{n}\int_0^{h_{1_i}(\mathbf{x})}\mathrm{Sat}\left[k_{p_i}\mathrm{Sat}(k_{p_{c_i}}r_i)+x_i+g_i(\mathbf{q}_d)\right]dr_i\\ &-\mathscr{U}(\mathbf{q}_d-\mathbf{h}_1(\mathbf{x})),\end{aligned} \tag{32}$$

where the last two terms are added in order to get $W(\mathbf{0},\mathbf{0})=0$.

By following similar steps to those given in [17] we prove that (32) is a positive definite and radially unbounded function, provided that $\lambda_{\min}\{K_pK_{p_c}\}>k_g$.

The time derivative of $W(\tilde{\mathbf{q}},\dot{\mathbf{q}})$ along the trajectories of (31) results in:

$$\begin{aligned}\dot{W}(\tilde{\mathbf{q}},\dot{\mathbf{q}})=&\,\dot{\mathbf{q}}^TM(\mathbf{q})\ddot{\mathbf{q}}+\frac{1}{2}\dot{\mathbf{q}}^T\dot{M}(\mathbf{q})\dot{\mathbf{q}}\\ &-\dot{\mathbf{q}}^T\mathbf{Sat}\left[K_p\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}})+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\right]+\mathbf{g}(\mathbf{q})^T\dot{\mathbf{q}}\\ =&\,\dot{\mathbf{q}}^T\left[\mathbf{Sat}\left[K_p\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}})-K_v\dot{\mathbf{q}}+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\right]\right.\\ &\left.-C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}-\mathbf{g}(\mathbf{q})\right]+\frac{1}{2}\dot{\mathbf{q}}^T\dot{M}(\mathbf{q})\dot{\mathbf{q}}\\ &-\dot{\mathbf{q}}^T\mathbf{Sat}\left[K_p\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}})+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\right]+\mathbf{g}(\mathbf{q})^T\dot{\mathbf{q}}\\ =&\,\dot{\mathbf{q}}^T\mathbf{Sat}\left[K_p\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}})-K_v\dot{\mathbf{q}}+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\right]\\ &-\dot{\mathbf{q}}^T\mathbf{Sat}\left[K_p\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}})+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\right].\end{aligned} \tag{33}$$

Finally, by using the following property of the saturation functions defined in Definition 2 (see Appendix A):

$$\dot{q}_i[\mathrm{Sat}(z_i-\dot{q}_i)-\mathrm{Sat}(z_i)]\le-|\mathrm{Sat}(z_i-\dot{q}_i)-\mathrm{Sat}(z_i)|^2 .$$

We have that $\dot{W}(\tilde{\mathbf{q}},\dot{\mathbf{q}})$ is upper bounded by:

$$\begin{aligned}\dot{W}(\tilde{\mathbf{q}},\dot{\mathbf{q}})\le-\Big\|&\mathbf{Sat}\left[K_p\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}})-K_v\dot{\mathbf{q}}+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\right]\\ &-\mathbf{Sat}\left[K_p\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}})+\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\right]\Big\|^2\le 0,\end{aligned}$$

thus $\dot{W}(\tilde{\mathbf{q}},\dot{\mathbf{q}})$ is a negative semidefinite function and we can conclude stability of the equilibrium point $[\tilde{\mathbf{q}}^T \ \ \dot{\mathbf{q}}^T]^T = [\mathbf{h}_1(\mathbf{x})^T \ \ \mathbf{0}^T]^T \in \mathbb{R}^{2n}$ of (31). We can use LaSalle's Invariance Principle to conclude that the equilibrium point is, in fact, globally asymptotically stable [7]. To this end, let us define Ω as:

$$\begin{aligned}\Omega &= \{\tilde{\mathbf{q}}, \dot{\mathbf{q}} \in \mathbb{R}^n : \dot{W}(\tilde{\mathbf{q}}, \dot{\mathbf{q}}) = 0\} \\ &= \{\dot{\mathbf{q}} = \mathbf{0}, \tilde{\mathbf{q}} \in \mathbb{R}^n\}.\end{aligned}$$

Notice that, from (31):

$$\begin{aligned}&\dot{\mathbf{q}}(t) \equiv \mathbf{0} \Rightarrow \ddot{\mathbf{q}}(t) \equiv \mathbf{0} \\ &\qquad \Rightarrow \mathbf{Sat}[K_p \mathbf{Sat}(K_{p_c} \tilde{\mathbf{q}}) + \mathbf{x} + \mathbf{g}(\mathbf{q}_d)] - \mathbf{g}(\mathbf{q}_d - \tilde{\mathbf{q}}) \equiv \mathbf{0}.\end{aligned}$$

Furthermore, under assumptions (21) and (23) we can assure that

$$\begin{aligned}&\mathbf{Sat}[K_p \mathbf{Sat}(K_{p_c} \tilde{\mathbf{q}}) + \mathbf{x} + \mathbf{g}(\mathbf{q}_d)] \\ &\qquad - \mathbf{g}(\mathbf{q}_d - \tilde{\mathbf{q}}) \equiv \mathbf{0} \Rightarrow \tilde{\mathbf{q}} \equiv \mathbf{h}_1(\mathbf{x}).\end{aligned}$$

Therefore, from LaSalle's Invariance Principle we conclude that the equilibrium point $[\tilde{\mathbf{q}}^T \ \ \dot{\mathbf{q}}^T]^T = [\mathbf{h}_1(\mathbf{x})^T \ \ \mathbf{0}^T]^T \in \mathbb{R}^{2n}$ of (31) is globally asymptotically stable.

5.1.2 Local exponential stability analysis

Before proceeding with the stability analysis of this section, we recall a useful existing lemma presented in [3].

Lemma 1: Consider the nonlinear system:

$$\dot{\mathbf{y}} = A(\mathbf{y})\mathbf{y} + B(\mathbf{y})\mathbf{f}(\mathbf{y}), \tag{34}$$

where $\mathbf{y} \in \mathbb{R}^m$, $A(\mathbf{y})$ and $B(\mathbf{y})$ are $m \times m$ nonlinear functions of $\mathbf{y}$, and $\mathbf{f}(\mathbf{y})$ is a $m \times 1$ nonlinear function of $\mathbf{y}$. Assume that $\mathbf{f}(\mathbf{0}) = \mathbf{0}$; hence, $\mathbf{y} = \mathbf{0} \in \mathbb{R}^m$ is an equilibrium point of the system (34). Then, the linearized system of (34) around the equilibrium $\mathbf{y} = \mathbf{0}$ is given by

$$\dot{\mathbf{y}} = \left[A(\mathbf{0}) + B(\mathbf{0}) \frac{\partial \mathbf{f}(\mathbf{0})}{\partial \mathbf{y}} \right] \mathbf{y}. \tag{35}$$

In order to prove that the equilibrium point of the closed-loop system (31) is locally exponentially stable, we consider a local linearization of the closed-loop system around the equilibrium point $[\tilde{\mathbf{q}}^T \ \ \dot{\mathbf{q}}^T]^T = [\mathbf{h}_1(\mathbf{x})^T \ \ \mathbf{0}^T]^T \in \mathbb{R}^{2n}$ [3]. In the neighborhood of this equilibrium point, the closed-loop system (31) can be represented by:

$$\begin{aligned}&M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) - K_p K_{p_c} \tilde{\mathbf{q}} \\ &\qquad + K_v \dot{\mathbf{q}} - \mathbf{x} - \mathbf{g}(\mathbf{q}_d) = \mathbf{0}.\end{aligned} \tag{36}$$

A local change of variables:

$$\begin{aligned}\mathbf{y}_1 &= \tilde{\mathbf{q}} - \mathbf{h}_1(\mathbf{x}), \\ \mathbf{y}_2 &= \dot{\mathbf{q}},\end{aligned}$$

leads to:

$$\begin{aligned}&M(\mathbf{q}_d - \mathbf{y}_1 - \mathbf{h}_1(\mathbf{x}))\dot{\mathbf{y}}_2 + C(\mathbf{q}_d - \mathbf{y}_1 - \mathbf{h}_1(\mathbf{x}), \mathbf{y}_2)\mathbf{y}_2 \\ &\quad + \mathbf{g}(\mathbf{q}_d - \mathbf{y}_1 - \mathbf{h}_1(\mathbf{x})) - K_p K_{p_c}[\mathbf{y}_1 + \mathbf{h}_1(\mathbf{x})] \\ &\quad + K_v \mathbf{y}_2 - \mathbf{x} - \mathbf{g}(\mathbf{q}_d) = \mathbf{0},\end{aligned}$$

whose unique equilibrium is the origin, provided that (23) is satisfied. The previous equation can be written as:

$$\dot{\mathbf{y}} = A(\mathbf{y})\mathbf{y} + B(\mathbf{y})\mathbf{f}(\mathbf{y}), \tag{37}$$

where

$$\dot{\mathbf{y}} = \frac{d}{dt}\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix},$$

$$A(\mathbf{y}) = \begin{bmatrix} 0 & -\mathbf{I} \\ 0 & -M(\mathbf{q}_d - \mathbf{y}_1 - \mathbf{h}_1(\mathbf{x}))^{-1}[K_v + C(\mathbf{q}_d - \mathbf{y}_1 - \mathbf{h}_1(\mathbf{x}), \mathbf{y}_2)] \end{bmatrix},$$

$$B(\mathbf{y}) = \begin{bmatrix} 0 & 0 \\ 0 & M(\mathbf{q}_d - \mathbf{y}_1 - \mathbf{h}_1(\mathbf{x}))^{-1} \end{bmatrix},$$

$$\mathbf{f}(\mathbf{y}) = \begin{bmatrix} \mathbf{0} \\ K_p K_{p_c}[\mathbf{y}_1 + \mathbf{h}_1(\mathbf{x})] + \mathbf{x} + \mathbf{g}(\mathbf{q}_d) - \mathbf{g}(\mathbf{q}_d - \mathbf{y}_1 - \mathbf{h}_1(\mathbf{x})) \end{bmatrix}.$$

According to Lemma 1, the linearized system of (37) around the equilibrium $\mathbf{y} = \mathbf{0}$ results in (35), with:

$$A(\mathbf{0}) = \begin{bmatrix} 0 & -\mathbf{I} \\ 0 & -M(\mathbf{q}_d - \mathbf{h}_1(\mathbf{x}))^{-1} K_v \end{bmatrix},$$

$$B(\mathbf{0}) = \begin{bmatrix} 0 & 0 \\ 0 & -M(\mathbf{q}_d - \mathbf{h}_1(\mathbf{x}))^{-1} \end{bmatrix},$$

$$\frac{\partial \mathbf{f}(\mathbf{0})}{\partial \mathbf{y}} = \begin{bmatrix} 0 & 0 \\ K^* & 0 \end{bmatrix},$$

which can be compacted in:

$$\frac{d}{dt}\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\mathbf{I} \\ M(\mathbf{q}_d - \mathbf{h}_1(\mathbf{x}))^{-1} K^* & -M(\mathbf{q}_d - \mathbf{h}_1(\mathbf{x}))^{-1} K_v \end{bmatrix}}_{J} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix}, \tag{38}$$

where K^* is given by:

$$K^* = K_p K_{p_c} - \frac{\partial \mathbf{g}(\mathbf{q}_d - \tilde{\mathbf{q}})}{\partial \mathbf{y}_1}.$$

Notice that, if (23) is satisfied, then K^* is a positive definite matrix.

To analyze the stability of the origin of (38) we propose the Lyapunov function candidate:

$$W_L(\mathbf{y}_1, \mathbf{y}_2) = \frac{1}{2}\mathbf{y}_2^T M(\mathbf{q}_d - \mathbf{h}_1(\mathbf{x}))\mathbf{y}_2 + \frac{1}{2}\mathbf{y}_1^T K^* \mathbf{y}_1, \tag{39}$$

which is a positive definite function. The time derivative along the trajectories of (38) is given by:

$$\begin{aligned}\dot{W}_L(\mathbf{y}_1,\mathbf{y}_2) &= \mathbf{y}_2^T M(\mathbf{q}_d-\mathbf{h}_1(\mathbf{x}))\dot{\mathbf{y}}_2+\mathbf{y}_1^T K^*\dot{\mathbf{y}}_1\\ &= \mathbf{y}_2^T[K^*\mathbf{y}_1-K_v\mathbf{y}_2]-\mathbf{y}_1^T K^*\mathbf{y}_2\\ &= -\mathbf{y}_2^T K_v\mathbf{y}_2,\end{aligned}$$

which is a negative semidefinite function. By using LaSalle's Invariance Principle we can conclude global asymptotic stability of the closed-loop system (38). To this end, let us define Ω as:

$$\begin{aligned}\Omega &= \{\mathbf{y}_1,\mathbf{y}_2\in\mathbb{R}^n: \dot{W}_L(\mathbf{y}_1,\mathbf{y}_2)=0\}\\ &= \{\mathbf{y}_2=\mathbf{0},\mathbf{y}_1\in\mathbb{R}^n\}.\end{aligned}$$

Notice that, from (38):

$$\begin{aligned}\mathbf{y}_2(t)\equiv\mathbf{0} &\Rightarrow \dot{\mathbf{y}}_2(t)\equiv\mathbf{0}\\ &\Rightarrow M(\mathbf{q}_d-\mathbf{h}_1(\mathbf{x}))^{-1}K^*\mathbf{y}_1\equiv\mathbf{0}.\end{aligned}$$

Furthermore, under assumption (23) we can assure that

$$M(\mathbf{q}_d-\mathbf{h}_1(\mathbf{x}))^{-1}K^*\mathbf{y}_1\equiv\mathbf{0}\Rightarrow\mathbf{y}_1\equiv\mathbf{0}.$$

Therefore, from LaSalle's Invariance Principle we conclude that the origin of the linear system (38) is globally asymptotically stable. This implies that the eigenvalues of J in (38) are located in the left-hand side of the complex plane (see [34], Theorem 4.5, pp. 134), and hence, the origin of the linear system (38) is exponentially stable (see e.g., [34], Theorem 4.11, pp. 156 that shows that for linear systems, uniform asymptotic stability of the origin is equivalent to exponential stability). According to this, exponential stability of the origin for the linear system (38) implies the local exponential stability of the origin for the nonlinear system (37) (see e.g. [34], Theorem 4.13, pp. 161).

Finally, we can conclude that the equilibrium point of the nonlinear system (31) is locally exponentially stable. So we have proven the following:

Proposition 1: By considering (12), (21), and (23), the control law (30) guarantees global asymptotic stability and local exponential stability of the closed-loop system (31) with $|\tau_i(t)|\le\tau_i^{\max}$, for all $i=1,2,\ldots,n$, and $t\ge 0$.

5.2. Stability analysis of the singularly perturbed system

To prove the exponential stability of the origin of (17)-(18), we recall an existing theorem.

Theorem 1 [34]**:** Consider the singularly perturbed system

$$\dot{\mathbf{x}}=\mathbf{f}(t',\mathbf{x},\mathbf{z},\varepsilon), \tag{40}$$

$$\varepsilon\dot{\mathbf{z}}=\mathbf{g}(t',\mathbf{x},\mathbf{z},\varepsilon). \tag{41}$$

Assume that the following assumptions are satisfied for all $(t',\mathbf{x},\varepsilon)\in\left[0,\infty\right)\times B_r\times[0,\varepsilon]$, with $B_r=\{\mathbf{x}\in\mathbb{R}^n:\|\mathbf{x}\|\le r\}$:

a) $\mathbf{f}(t',\mathbf{0},\mathbf{0},\varepsilon)=\mathbf{0}$ and $\mathbf{g}(t',\mathbf{0},\mathbf{0},\varepsilon)=\mathbf{0}$.

b) The equation $\mathbf{0}=\mathbf{g}(t',\mathbf{x},\mathbf{z},\varepsilon)$ has an isolated root $\mathbf{z}=\mathbf{h}(t',\mathbf{x})$ such that $\mathbf{h}(t',\mathbf{0})=\mathbf{0}$.

c) The functions $\mathbf{f}$, $\mathbf{g}$, $\mathbf{h}$, and their partial derivatives up to the second order are bounded for

$$\mathbf{z}-\mathbf{h}(t',\mathbf{x})\in B_\rho,\ \text{ with }\ B_\rho=\{\mathbf{y}\in\mathbb{R}^{2n}:\|\mathbf{y}\|\le\rho\}.$$

d) The origin of the reduced system

$$\dot{\mathbf{x}}=\mathbf{f}(t',\mathbf{x},\mathbf{h}(t',\mathbf{x}),0) \tag{42}$$

is exponentially stable.

e) The origin of the boundary-layer system

$$\frac{d\mathbf{y}}{dt}=\mathbf{g}(t',\mathbf{x},\mathbf{y}+\mathbf{h}(t',\mathbf{x}),0) \tag{43}$$

is exponentially stable, uniformly in $(t',\mathbf{x})$.

Then, there exists $\varepsilon^*>0$ such that for all $\varepsilon<\varepsilon^*$, the origin of (40)-(41) is exponentially stable.

We are now ready to present our main contribution.

Proposition 2: Consider the robot dynamics (1) in closed-loop with the practical saturated PID control law (11). Under Assumption 2, (21), (23) and $\lambda_{\min}\{K_{p_c}\}>k_h$, with k_h given by $k_h=\dfrac{2k'}{\mathrm{Sat}\left(\dfrac{2k'\lambda_{\min}\{K_p\}}{k_g}\right)}$, the origin of the closed-loop system (17)-(18) is locally exponentially stable, and therefore, the equilibrium point of (13) is locally exponentially stable. Besides, $|\tau_i(t)|\le\tau_i^{\max}$, for all $i=1,2,\ldots,n$ and $t\ge 0$.

Proof: Notice that (17)-(18) correspond to (40)-(41), with

$$\mathbf{f}(t',\mathbf{x},\mathbf{z},\varepsilon)=K_i^*\left[\alpha\mathbf{Sat}(K_{p_c}\tilde{\mathbf{q}})-\dot{\mathbf{q}}\right],$$

$$\mathbf{g}(t',\mathbf{x},\mathbf{z},\varepsilon)=\begin{bmatrix}-\dot{\mathbf{q}}\\ M(\mathbf{q})^{-1}\Big[\mathbf{Sat}\Big[K_p\mathbf{Sat}\big(K_{p_c}\tilde{\mathbf{q}}\big)-K_v\dot{\mathbf{q}}\\ +\mathbf{x}+\mathbf{g}(\mathbf{q}_d)\Big]-C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}-\mathbf{g}(\mathbf{q})\Big]\end{bmatrix},$$

$$\mathbf{z}=\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\end{bmatrix}\in\mathbb{R}^{2n}.$$

In order to complete the stability analysis, we are going to check each item of Theorem 1.

a) By substituting $\mathbf{x}=\tilde{\mathbf{q}}=\dot{\mathbf{q}}=\mathbf{0}$ in (17)-(18), it is straightforward to verify this assumption.

b) This item is easily fulfilled by noting that the root of $\mathbf{g}(t',\mathbf{x},\mathbf{z},\varepsilon)$ has been obtained in Subsection 4.2., where it was proven that, for each $\mathbf{x}\in\mathbb{R}^n$, the unique root of (18) is $\mathbf{z}=\mathbf{h}(\mathbf{x})=[\mathbf{h}_1(\mathbf{x})^T\ \ \mathbf{0}^T]^T\in\mathbb{R}^{2n}$, provided that (23) is satisfied. On the other hand we know, from (24), that $\tilde{\mathbf{q}} = \mathbf{h}_1(\mathbf{x})$ and, therefore, when $\mathbf{x} = \mathbf{0}$ we have $\tilde{\mathbf{q}} = \mathbf{h}_1(\mathbf{0})$; then, from (25), $\mathbf{0} = \mathbf{h}_1^{-1}(\tilde{\mathbf{q}}) = -[K_p K_{pc} \tilde{\mathbf{q}} + \mathbf{g}(\mathbf{q}_d) - \mathbf{g}(\mathbf{q}_d - \tilde{\mathbf{q}})]$ which, under assumption (23), has a unique solution $\tilde{\mathbf{q}} = \mathbf{0}$. Hence, $\mathbf{h}(\mathbf{0}) = [\mathbf{h}_1(\mathbf{0})^T \quad \mathbf{0}^T]^T = [\mathbf{0}^T \quad \mathbf{0}^T]^T$ and therefore assumption b) is verified.

c) This is straightforward because the right hand-side of (17)-(18) is C^2.

d) By substituting the isolated root $\mathbf{z} = \mathbf{h}(\mathbf{x})$ and $\varepsilon = 0$ in (17), that is $\tilde{\mathbf{q}} = \mathbf{h}_1(\mathbf{x})$ and $\dot{\mathbf{q}} = \mathbf{0}$, we obtain the so called reduced system, which is given by:

$$\frac{d}{dt'}\mathbf{x} = K_i^* \left[\alpha \, \mathbf{Sat}\left(K_{p_c} \mathbf{h}_1(\mathbf{x}) \right) \right], \tag{44}$$

whose unique equilibrium point results from $\mathbf{h}_1(\mathbf{x}) = \mathbf{0}$ and is given by $\mathbf{x} = \mathbf{h}_1^{-1}(\mathbf{0}) = \mathbf{0}$, provided that (23) is satisfied. Comparing the reduced system (44) with the terms used in Theorem 1, we have

$$\dot{\mathbf{x}} = \mathbf{f}(t', \mathbf{x}, \mathbf{h}(t, \mathbf{x}), 0) = K_i^* [\alpha \, \mathbf{Sat}(K_{p_c} \mathbf{h}_1(\mathbf{x}))].$$

On the other hand, to analyze the origin of the reduced system (44), let us define the quadratic Lyapunov function candidate

$$V(\mathbf{x}) = \frac{1}{2} \mathbf{x}^T (K_i^*)^{-1} \mathbf{x}, \tag{45}$$

which satisfies

$$V(\mathbf{x}) \geq \lambda_{\min}\{(K_i^*)^{-1}\} \, \|\mathbf{x}\|^2 \tag{46}$$

and hence, is a positive definite and radially unbounded function. The time derivative along the trajectories of (44) is given by

$$\begin{aligned} \dot{V}(\mathbf{x}) &= \mathbf{x}^T (K_i^*)^{-1} \dot{\mathbf{x}} \\ &= \alpha \, \mathbf{x}^T \mathbf{Sat}(K_{p_c} \mathbf{h}_1(\mathbf{x})) \\ &\leq -\alpha \left[\lambda_{\min}\{K_p\} - k_h \right] \left\| \mathbf{Sat}(K_{p_c} \mathbf{h}_1(\mathbf{x})) \right\|^2, \end{aligned} \tag{47}$$

where we have used the fact that $\mathbf{x}^T \mathbf{Sat}(K_{p_c} \mathbf{h}_1(\mathbf{x})) \leq -\left[\lambda_{\min}\{K_p\} - k_h \right] \left\| \mathbf{Sat}(K_{p_c} \mathbf{h}_1(\mathbf{x})) \right\|^2$ with k_h given by ([7], pp 108):

$$k_h = \frac{2k'}{\mathbf{Sat}\left(\dfrac{2k' \lambda_{\min}\{K_{p_c}\}}{k_g} \right)}.$$

Notice that, due to $\mathbf{h}_1(\mathbf{0}) = \mathbf{0}$, the time derivative (47) is a negative definite function provided that

$$\lambda_{\min}\{K_p\} > k_h, \tag{48}$$

and so we can conclude global asymptotic stability of the origin of (44).

In order to prove local exponential stability of the origin of (44), observe that (25) can be locally approximated by:

$$\mathbf{x} = -K_p K_{p_c} \mathbf{h}_1(\mathbf{x}) - \mathbf{g}(\mathbf{q}_d) + \mathbf{g}(\mathbf{q}_d - \mathbf{h}_1(\mathbf{x})).$$

Furthermore, we can get:

$$\|\mathbf{x}\|^2 \leq [\lambda_{\max}\{K_{p_c}\} \lambda_{\max}\{K_p\} + k_g]^2 \, \|\mathbf{h}_1(\mathbf{x})\|^2.$$

So, from the last equation, we obtain:

$$-\|\mathbf{h}_1(\mathbf{x})\|^2 \leq -\frac{1}{[\lambda_{\max}\{K_{p_c}\} \lambda_{\max}\{K_p\} + k_g]^2} \|\mathbf{x}\|^2. \tag{49}$$

Then, by substituting (49) in the local approximation of (47), we obtain:

$$\begin{aligned} \dot{V}(\mathbf{x}) &\leq -\alpha \left[\lambda_{\min}\{K_p\} - k_h \right] \left\| K_{p_c} \mathbf{h}_1(\mathbf{x}) \right\|^2 \\ &\leq -\alpha \lambda_{\min}\{K_{p_c}\}^2 \left[\lambda_{\min}\{K_p\} - k_h \right] \left\| \mathbf{h}_1(\mathbf{x}) \right\|^2 \\ &\leq -\frac{\alpha \lambda_{\min}\{K_{p_c}\}^2 \left[\lambda_{\min}\{K_p\} - k_h \right]}{\left[\lambda_{\max}\{K_{p_c}\} \lambda_{\max}\{K_p\} + k_g \right]^2} \|\mathbf{x}\|^2, \end{aligned}$$

where we have used the fact that $\left\| K_{p_c} \mathbf{h}_1(\mathbf{x}) \right\|^2 \geq \lambda_{\min}\{K_{p_c}\}^2 \left\| \mathbf{h}_1(\mathbf{x}) \right\|^2$.

Therefore, we can conclude that $\mathbf{x} = \mathbf{0}$ is a locally exponentially stable equilibrium point for the reduced system (44), provided that $\lambda_{\min}\{K_p\} > k_h$. So we have verified the assumption d) of Theorem 1.

e) By setting $\varepsilon = 0$ and considering that $\varepsilon \dfrac{d\mathbf{y}}{dt'} = \dfrac{d\mathbf{y}}{dt}$ in (28), we obtain the boundary-layer system, i.e.,

$$\underbrace{\frac{d}{dt} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix}}_{\frac{d\mathbf{y}}{dt}} = \underbrace{\begin{bmatrix} -\mathbf{y}_2 \\ M(\mathbf{q}_d - \mathbf{y}_1 - \mathbf{h}_1(\mathbf{x}))^{-1} \Big[\mathbf{Sat} \\ \left[K_p \mathbf{Sat}\left(K_{p_c} \left(\mathbf{y}_1 + \mathbf{h}_1(\mathbf{x}) \right) \right) - K_v \mathbf{y}_2 + \mathbf{x} + \mathbf{g}(\mathbf{q}_d) \right] \\ -C(\mathbf{q}_d - \mathbf{y}_1 - \mathbf{h}_1(\mathbf{x}), \mathbf{y}_2) \mathbf{y}_2 - \mathbf{g}(\mathbf{q}_d - \mathbf{y}_1 - \mathbf{h}_1(\mathbf{x})) \Big] \end{bmatrix}}_{\mathbf{g}(t, \mathbf{x}, \mathbf{y} + \mathbf{h}(t, \mathbf{x}), 0)}, \tag{50}$$

where, according to (29), $\mathbf{x}$ is frozen at $\mathbf{x} = \mathbf{x}(t_0)$, which corresponds to the robotic system under the Saturated PD Controller with Desired Gravity Compensation plus a constant vector $\mathbf{x}$, whose unique equilibrium point is the origin, provided that $\lambda_{\min}\{K_p K_{p_c}\} > k_g$ is satisfied.

The stability analysis of (50) has already been carried out in the previous subsection, where we concluded, in accordance with Proposition 1, that the origin of (50) is asymptotically stable and locally exponentially stable, uniformly in $\mathbf{x}$. The uniformity in $\mathbf{x}$ is given straighfor-

Table 1. Values of the selected control pararameters.

Gain	Joint 1	Joint 2	Units
K_p	20.0	4.0	[Nm/deg]
K_v	2.5	0.5	[Nm · s/deg]
K_i	0.2	0.02	[Nm/deg]
K_{p_c}	1.58	1.16	
$\mathbf{l}_p$	97	8.5	[Nm]
$\mathbf{m}_p$	98	9	[Nm]
$\mathbf{l}_{pi}$	99	9.5	[Nm]
$\mathbf{m}_{pi}$	100	10	[Nm]

ward with the asymptotic stability of the origin of (50) because it is an autonomous system. This checks the assumption e).

Finally, we can conclude, in accordance with Theorem 1, that the equilibirum point of the closed-loop system (13) is locally exponentially stable for a sufficiently small ε. Under Assumption 2, constraints (5) are trivially satisfied. This completes the proof.

Remark 2: The computation of the boundary of ε is in general a difficult task [19,20]. Proposition 2 implies the existence of a positive constant $\varepsilon^* > \varepsilon > 0$ such that the singularly perturbed system (17)-(18) is semiglobally asymptotically and locally exponentially stable about the origin of the closed-loop system for small enough integral gain $K_i = \varepsilon K_i^*$.

6. EXPERIMENTAL RESULTS

In order to illustrate the stability results described in the previous pages, this section shows a real-time experimental essay on the two-revolute-joint direct-drive manipulator presented in [35,36].

The aim of the proposal introduced in this paper is not to improve the performance of the saturated PID controllers previously presented in the literature, but to justify theoretically why the classical linear PID controller, used in industrial robot manipulators, works well in practice, in spite of entering inside of the saturation zones of the actuators and/or the electronic control devices. However, with the end of having a fair comparison frame, we have chosen the same operational conditions used in the experimental essays reported in [20]; that is, the desired joint positions were chosen as $q_{d1} = 45^\circ$ and $q_{d2} = 90^\circ$. Property 2 is satisfied with $\gamma_1(\mathbf{q}) = 40.289$Nm, $\gamma_2(\mathbf{q}) = 1.827$Nm and $k_g = 1.406$ Nm/deg. The parameters of the controller were adjusted to the values shown in Table 1 with α =8s^{-1}. It can be easily verified that for $i = 1,2$, $\tau_i^{\max} > \gamma_i(\mathbf{q})$. The integral gains were chosen sufficiently small to achieve a good response. Fig. 2 shows no overshoot for the joint 1 and a small overshoot for the joint 2.

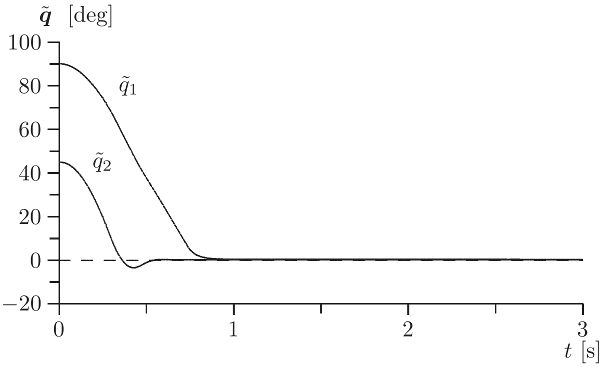

Fig. 2. Joint position errors for the practical PID controller with bounded torques.

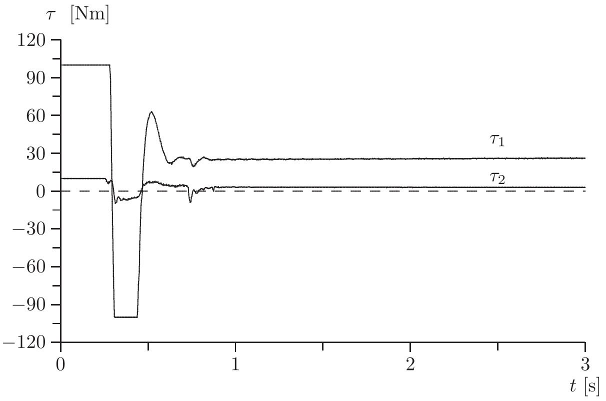

Fig. 3. Applied torques using the practical PID controller with bounded torques.

Fig. 2 shows how the position errors converge to zero and Fig. 3 shows the torques for a period of three seconds. We can observe in Fig. 3 that the proposed saturated nonlinear PID controller supplies control torques satisfying $|\tau_1| < \tau_1^{\max}$ =150Nm, and $|\tau_2| < \tau_2^{\max}$ =15Nm. Notice that, by safety, the actuator torques are intentionally saturated to smaller values than the maximum torques provided by the manufacturer.

7. CONCLUSIONS

In this paper we proposed a saturated nonlinear PID controller which, in fact, results from the practical implementation of the classical PID controller, considering the natural saturations of the control computer and those of the servo-drivers and actuators. The stability analysis of the closed-loop system was carried out by using the singular perturbations theory. Based on auxiliary Lyapunov functions we proved local exponential stability of the equilibrium point of the closed-loop system.

It is also guaranteed that, regardless of initial conditions, the delivered actuator torques evolve inside permitted limits. Experimental results confirmed the proposed analysis.

Furthermore, the theoretical result explains why the classical linear PID regulator used in industrial robot manipulators preserves the exponential stability in spite of entering into the saturation zones inherent to the electronic control devices and the actuator torque constraints.

APPENDIX A

Due to $\mathrm{Sat}(\cdot)$ is a strictly increasing function (see Definition 2), we have that, for the scalar case [17]:

$$\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)>0 \Leftrightarrow z-\dot{q}>z, \tag{51}$$

which is satisfied if and only if $\dot{q}<0$. Therefore,

$$\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)>0 \Leftrightarrow \dot{q}<0. \tag{52}$$

For the opposite case of (51), we have that:

$$\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)<0 \Leftrightarrow z-\dot{q}<z,$$

that is, if and only if $\dot{q}>0$. So that,

$$\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)<0 \Leftrightarrow \dot{q}>0. \tag{53}$$

Using (52) and (53) we can conclude that

$$\dot{q}\left[\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)\right]<0\,\forall\, z,\dot{q}\in\mathbb{R}.$$

On the other hand, we know that (see pp. 91 of [34]):

$$\left|\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)\right|\le\left|z-\dot{q}-z\right|=\left|\dot{q}\right| \quad \forall\, z,\dot{q}\in\mathbb{R}. \tag{54}$$

We have that $-\dot{q}$ has the same sign of $\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)$, then we can write

$$\begin{aligned}-\dot{q}\left[\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)\right]&=\left|-\dot{q}\right|\left|\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)\right|\\&=\left|\,\dot{q}\right|\left|\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)\right|\\&\ge\left|\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)\right|^{2}\end{aligned}$$

where we have used (54). Finally we get

$$\dot{q}\left[\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)\right]\le-\,|\,\mathrm{Sat}(z-\dot{q})-\mathrm{Sat}(z)\,|^{2}\,.$$

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